

ANA, the inverse problem and the Zero Dipole Localization Error

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Definition of ANA: the simplest inverse solution with nearly ideal Resolution Matrix

Given the inverse problem for J: $V=LJ$
define the inverse as the adjoint (transpose) of the column normalized lead field L_N , i.e.,

$$G = (Lw^{-1})^t = (L_N)^t$$

where W_{ii} is the norm of the ith Lead Field Column and t denotes transpose.

Theoretical properties of ANA (Adjoint Normalized Approximation) inverse matrix

- Symmetric resolution matrix (in L_N) with ones in the diagonal (unitary gain) what implies:
- Perfect reconstruction of position and amplitudes of all single sources (for the first time!!)
- Resolution kernels and point spread functions peaking at the diagonal elements of the resolution matrix.

Symmetric resolution matrix with unitary diagonal elements (in L_N)

1.00	0.48	0.94	0.48	-0.84	-0.75	-0.67	-0.86	-0.93	-0.10	-0.93	0.16
	1.00	0.74	0.99	-0.87	-0.94	-0.97	-0.85	-0.13	0.81	-0.14	0.94
		1.00	0.75	-0.97	-0.92	-0.88	-0.98	-0.75	0.23	-0.76	0.48
			1.00	-0.87	-0.94	-0.97	-0.85	-0.14	0.81	-0.15	0.94
				1.00	0.98	0.96	0.99	0.60	-0.43	0.61	-0.66
					1.00	0.99	0.98	0.46	-0.57	0.47	-0.77
						1.00	0.95	0.36	-0.66	0.37	-0.83
							1.00	0.62	-0.40	0.63	-0.63
								1.00	0.45	0.99	0.20
									1.00	0.44	0.96
										1.00	0.19
											1.00

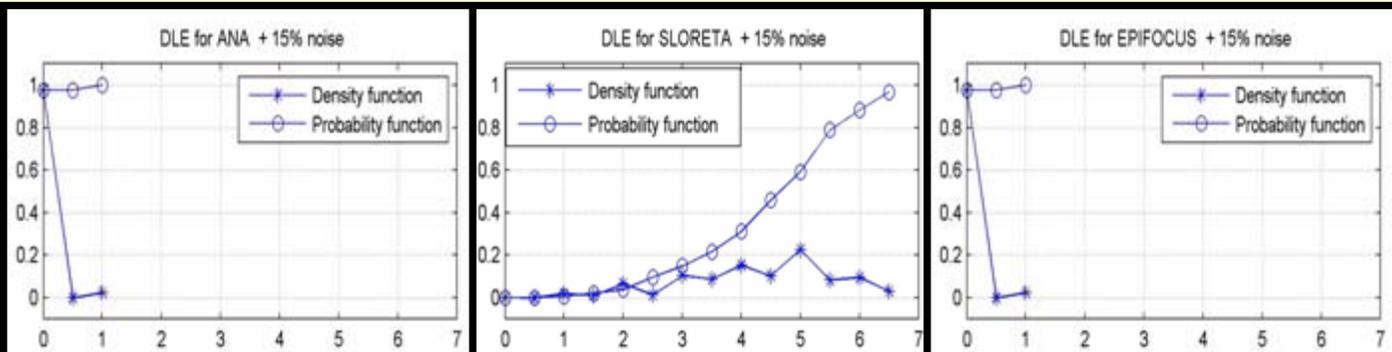
← Simulation with 2 Electrodes and 4 solution points
Note max always are always in the diagonal →

The reconstruction of two sources (sum of rows 1 and 12) attains the maximum (abs) at the 6th coordinate that does not corresponds to any of the active points (see conclusion 2):

[1.2, 1.42, 1.42, 1.43, -1.50, -1.52, -1.51, -1.50, -0.72, 0.86, -0.74, 1.16]

Simulation results for 148 electrodes and 2451 single sources (817 solution points). **Random noise** with amplitude up to **15%** of the potential at each electrode. For the comparison we use 4 different regularization values for sLORETA (low=0, medium=0.1 and 1, high=10) but all yielded similar results.

Proportion of correctly localized sources (from 0 to 1) vs. distance to the target source (in original space L)



Conclusions: The simulations presented in this paper demonstrates that:

1. ANA inverse solution provides **excellent localization** results in both the transformed and the original spaces and all this with a **minimal computational complexity**, **nevertheless**
2. The perfect identification of single sources (in location and magnitude!) **is not even sufficient** for the **approximate localization of multiple sources** in the normalized space L_N .
3. For **realistic conditions** (data with time varying noise) ANA and EPIFOCUS produce the smaller **localization errors**.