Author version: Biomedizinische Technik, 42:53-56

A LINEAR INVERSE SOLUTION WITH OPTIMAL RESOLUTION PROPERTIES : WROP.

Grave de Peralta Menendez, R., Gonzalez Andino, S. L., Hauk*, O., Spinelli, L.,Michel, C.M.

Brain Mapping Lab. Geneva University Hospital, 1211 Geneva 14, Switzerland.
*Department of Clinical Neurophysiology, Magdeburg, FRG

Introduction

Linear inverse solutions have been extensively applied in the bioelectromagnetic inverse problem to yield a three dimensional reconstruction of the current distribution within the human brain [1]. In order to insure a unique solution, the linear methods proposed so far, minimize some predefined global property that the actual current distribution is expected to fulfill, e.g., minimum norm, maximum smoothness. In contrast the Backus and Hilbert approach, recently applied for the first time to the estimation of vector fields [2], attempts to obtain the best possible resolution from a given set of data. This framework allows not only the analysis and comparison of linear inverse solutions [2] as well as the design of optimal sensor configurations but can also be used to derive solution with optimal resolution kernels [4]. The Weighted Resolution Optimization (WROP) method, that will be described in this paper, is based on this framework. In particular we will describe the theoretical basis and flexibility of the WROP method and present a comparison with other linear methods using a simple model. Finally we show the results obtained in the analysis of real data, i.e., visual evoked potentials, where some a priori knowledge about the generators is available. The paper finish with a brief discussion about the different alternatives and future trends.

Basic Theory

Consider the underdetermined linear problem with model matrix **L**, measured data vector **d**, unknown vector **j**, and additive noise **n**, such that:

$$\mathbf{d} = \mathbf{L}\mathbf{j} + \mathbf{n} \qquad (1)$$

building in linearity [3] the solution can be obtained in terms of the (inverse) matrix **G**, then

$$\hat{\mathbf{j}} = \mathbf{G}\mathbf{d} \qquad (2)$$

substituting **d** according to (1) in (2) we obtain the fundamental equation for underdetermined linear systems:

$$\hat{\mathbf{j}} = \mathbf{G}\mathbf{L}\mathbf{j} + \mathbf{G}\mathbf{n} = \mathbf{R}\mathbf{j} + \mathbf{n}' \qquad (3)$$

where **R=GL** is the resolution matrix [2]. In absence of noise, i.e., **n**=0, this matrix reveals the relationship between the estimates $\hat{\mathbf{j}}$ and any solution **j** of equation (1), i.e., any solution that produces the same data. When this correspondence is one to one, the inverse problem has a unique solution. This is the reason to consider **R** a "measure" of the uncertainty of the problem. When **R=I** (identity matrix) the estimated parameters coincide with the original ones. In general, the closer the resolution matrix to the identity matrix, the better is the resemblance between original and estimated parameters. On this basis the following measure of closeness between the resolution and the identity matrix can be optimized with respect to the rows of the inverse matrix **G** [4]:

$$\sum_{i,j} \mathbf{W}_{ij}(\mathbf{R}_{ij} - \mathbf{I}_{ij})^2 = \sum_i \mathbf{G}_{i.}^t \mathbf{L} \widetilde{\mathbf{W}}_{i.} \mathbf{L}^t \mathbf{G}_{i.} - 2\mathbf{W}_{ii}\mathbf{G}_{i.}^t \mathbf{L}_{.i} + \mathbf{W}_{ii} \qquad (4)$$

The following table summarizes the solutions that can be obtained by changing the weights, illustrating the flexibility of the WROP method.

$\mathbf{W}_{ij} = \text{constant } \forall ij$	Minimum Norm
$\mathbf{W}_{ij} = \mathbf{p}_j \;\; \forall ij$	Weighted Minimum Norm
$\mathbf{W}_{ij} = \mathbf{p}_j^i$	Weighted Minimum Norm with different weights for any point i

Table 1: Type of solutions obtained for the WROP method by changing the weights.

Strategies for the selection of the weights based on the distances between points were considered in [4]. There we showed that the WROP method with distance dependent weights can be considered as a regularization of the (weighted) minimum norm with the regularization term determined by the Backus and Gilbert spread. The relationships with other methods that optimize resolution are also considered, offering a general framework to cast these solutions.

Results

The analysis of distributed linear solutions poses a difficult problem. Before the proposals presented in [2] and [3], distributed inverse solutions have been mainly analyzed in terms of the dipole localization error (DLE): a measure derived from the pattern estimated (impulse response) when the data is generated by a dipolar source of unitary strength (delta function). This measure is dependent on the distance between the maximum of the map of the modules and the position of the dipolar source. It seems to be adequate in the case of overdetermined models, e.g., multiple dipole models, but does not provide real advantages in the case of underdetermined models, i.e., distributed inverse solutions. The main reasons for this are [5]: First that magnitudes like the module of a vector field, i.e., the DLE, are non-linear functions of the solution. Thus, superposition does not apply to them. On this basis, the knowledge of an inverse solution over a set of dipolar source does not allow to make inferences about more complex sources, i.e., linear combination of the simplest sources. Second in order to properly retrieve a source, an estimate of the location together with the amplitude of the source should be given. Clearly the DLE does not provide this information. Note that even if it is very easy to construct solutions with dipole localization errors independent of the depth (eccentricity) of the source [3], the same does not hold for the amplitude. The amplitude of deeper sources seems to be always underestimated [5].

For these reasons, we suggest to evaluate inverse solution properties with measures that are based on the resolution kernels, since they determine the relationship between the estimates produced by an inverse solution and the source distribution that produced the data (3).

In the following simulation we compare three linear inverse solutions with the WROP method on the basis of their resolution kernels. For the sake of simplicity, i.e., to obtain a clear representation of the resolution kernels, we will use a one dimensional model.

As stated in [2], [3] matrix **R** is an appropriate basis for the comparison of linear inverse solutions. In particular its rows, that are called resolution kernels, shed light on the way simultaneously active sources can affect the estimates in one point. According to its interpretation [2], [4] good resolution kernels are expected to peak at the target point with minimum sidelobes, which, if present, should be decreasing with the distance to the point.

Figure 1 depicts the resolution kernels associated to 4 type of linear inverse solutions, namely: Minimum Norm, Weighted minimum norm, Minimum Laplacian and WROP. A one dimensional model was selected with 11 sensors located over the upper half of the circle and 101 solution points on the diameter of the circle of radius 20. It was assumed that the direction of the field was known, as common in cortical reconstruction. Thus, only the amplitude should be determined for every point. The selected point corresponds to a "deep location", i.e., a location distant from the measurement surface. Note that for this point, marked with the vertical line, the best resolution kernel is found for WROP, since the resolution kernel has minimum sidelobes that approach zero for far away points. For all the other 3 solutions the resolution kernels show an oscillatory behavior all over the solution space. As a consequence, the estimates for the target point will be seriously affected for sources located on the points corresponding to the peaks of the sidelobes. This explains why spurious sources can appear in the reconstruction. WROP reduces these undesired effects but, in consequence, will be somewhat smoother, i.e., the reconstruction will be flatter around the target point. This "preventive smoothing" of the solution is based on the principle that details that are not imposed by the data should not appear in the reconstruction.

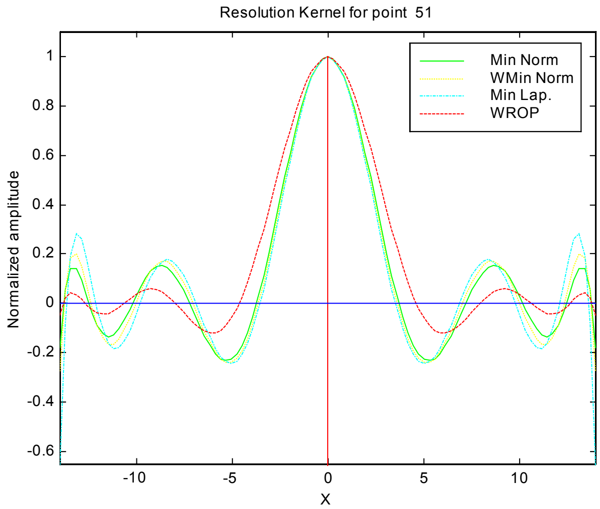

Figure 1. Resolution kernels for a deep point (center of the circle). Note the minimum of the sidelobes obtained for WROP in contrast to Minimum Norm, Weighted Minimum Norm and Minimum Laplacian (maximum smoothness).

Since the real scope of these methods is to extract information about how the brain works, the second part is devoted to the analysis of real data, i.e., visual evoked potentials, where some knowledge about the feasible location of the sources exists.

We applied the WROP method to checkerboard visual evoked potentials (VEP), recorded from 41 electrodes in 25 subjects. By asking the subjects to fixate on a cross that was either presented to the left, right or top of the figure, separate stimulation of the right or left visual field was compared with stimulation of both visual fields. The figure consisted of a black/white checkerboard pattern with a spatial frequency of 0.5 cycles/deg. Reversal of the pattern occurred every 500 ms. Hundred artifact free sweeps were averaged per subject and condition. Grand mean evoked potentials over subjects were calculated by aligning all individual VEPs to the strongest P100 map, i.e., to the maximal global field power peak at around 100 ms. This alignment diminishes temporal smearing produced by inter-individual variations. The peak latency was determined visually in each VEP trace in order to ensure that the map at this peak showed the known occipital positivity. Three time frames were selected in the grand mean VEPs, corresponding to the conventional N70, P100, and N140 components. In our data, these three components were found at 72, 105, 147 ms for left visual field stimulation, at 75, 109, and 153 ms for right visual field stimulation, and at 75, 106, and 160 ms for central stimulation. The WROP solutions of the grand mean data for these time points are shown in Figure 2 as slices through the brain at different heights. As expected from anatomico-physiological and electrophysiological data, WROP localized cerebral activation in the right occipital cortex for left visual field stimulation, and vice versa in the left occipital cortex after right visual field stimulation. This lateralized activity was found for all three peaks. Central stimulation resulted in strongest activity in the mesial occipital cortex for the three selected time points. A stronger activity in the right occipital cortex was found for time points before 160 ms.

Discussion

The solution of linear undetermined inverse problems is an ill-posed problem with infinite solutions. The only way to face the curse of the non-uniqueness is to add some a priori information to constraint the solution set. Such information might reflect our expectation about the sources and can be determined by a global property [1]. Another way to obtain a unique solution is optimizing a figure of merit like resolution [2], [4]. The WROP method presented here shows a possible way to improve the resolution properties based on the assumption that the optimal resolution kernel is the one closest to the ideal one. The comparison in the one dimensional example shows that the sidelobes can be reduced in the WROP method as compared to the minimum norm type methods where the influence of points in the boundary remains high.

Even if it cannot be considered as a validation, the agreement between the solution obtained in the case of evoked potentials with the physiological expectation is encouraging. Our experience from the analysis of experimental data is that all linear solutions produce very similar results independently of their particular behavior for unitary single sources. A good practical reason to avoid comparisons based on the DLE. However, slight differences can be found with solutions that include weights depending on the position of the sensors, i.e., depending on the columns of the model matrix **L**, especially for non-symmetric sensor configurations. Nevertheless, if the analysis is restricted to determine the quadrant of maximum activity (which is of practical interest for example in epilepsy), we found no difference between methods, including the Minimum Norm.

The application of inverse solution to real data face us with the two following problems:

a)which of the estimated sources are spurious (ghost sources).

b) which of the real sources were suppressed (lost sources).

So far there is no definitive answer to these questions thus, inferences about brain processes on the basis of estimated maps should be carefully considered. Based on several simulations we can say that there is no linear inverse solution able to properly retrieve single (dipolar) sources in the whole three-dimensional solution space. The problem is even worse for more complex source configurations. On the other hand the existence of a linear solution without ghost and lost sources seems to be mathematically impossible. This can be concluded from the analysis of the resolution matrix and justify the development of linear inverse solutions with optimal resolution kernels. On this direction methods like WROP could be useful.

Among the alternative solutions that deserve more attention in the future are those based on constraints in the source space, on this direction the constrained inverses [3] can be applied having in mind the kind of resolution kernels that can appear. Still in linearity, a characterization of the kind of source that generates the data [3] and/or the set of invisible objects could be an alternative to arrive to simpler problems, i.e., to reduce the number of unknowns and select alternatives models for the sources. Non-linear solutions need to be more explored. In particular, constraints on the temporal behavior including information about higher statistics in the time/frequency domain, combined with locally spatial constraints, e.g., autocorrelation should be considered.

Even when we should not expect miracles in the solution of this inverse problem an optimistic future can be expected if new information (linear or not) from other methods of study of the brain functioning is included.

References

[1] George, J.S., Aine, C.J., Mosher, J.C., Schmidt, D.M., Ranken, D.M., Schlitt, H.A., Wood, C.C., Lewine, J.D., Sanders, J.A. and Belliveau, J.W. Mapping function in the human brain with magneto-encephalography, anatomic MRI, and functional MRI. J. Clin. Neurophysiol. 1995, 12: 406-431.

[2] Grave de Peralta, R., Gonzalez, S., and Lütkenhöner, B., Figures of merit to compare linear distributed inverse solutions. Brain Topography. 1996, Vol. 9. 2:117-124.

[3] Grave de Peralta Menendez R and Gonzalez Andino SL, A critical analysis of linear inverse solutions. IEEE Trans. Biomed. Engn. 1996, (in process).

[4] Grave de Peralta, R., Hauk, O., Gonzalez, S., Vogt, H., Michel, C.M. Linear inverse solutions with optimal resolution kernels applied to the electromagnetic tomography. Human Brain Mapping. 1977, (in process).

[5] Grave de Peralta, R., Gonzalez, S., Michel, C.M., Does a linear tomographic solution exist?. In preparation.

.

Acknowledgements

This work was supported by the Swiss National Science Foundation (PNR38- 4038-044081/1).

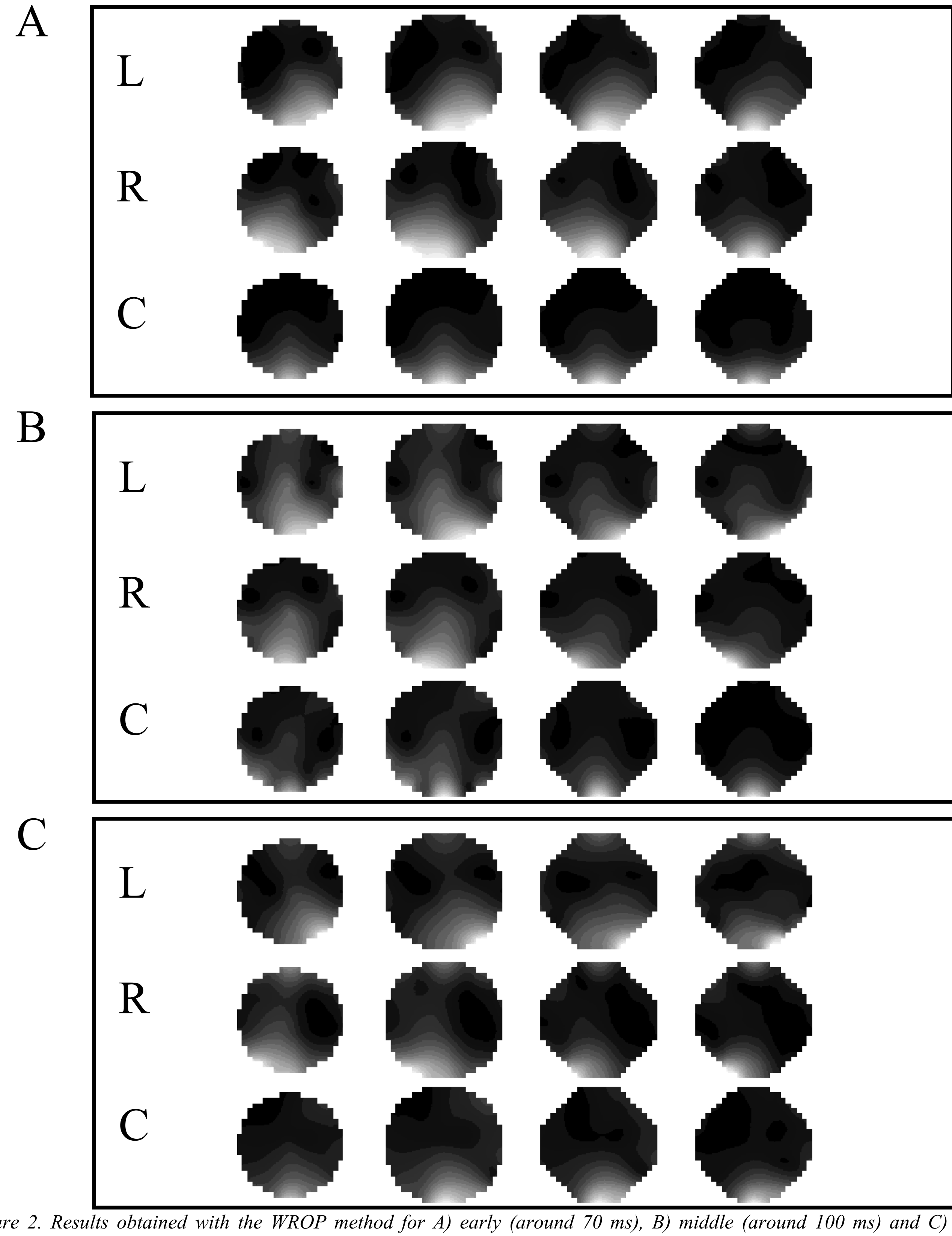

Figure 2. Results obtained with the WROP method for A) early (around 70 ms), B) middle (around 100 ms) and C) later (around 150 ms) components of visual evoked potentials. The rows correspond to left (L), right (R) and central (C) visual field stimulation. For more details see the text.