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The resolution-field concept

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Abstract

The concept of a resolution field provides a means to compare arbitrary estimators of a brain activity of interest (AOI), represented, for example, by the amplitude of a dipole at a certain location of interest with well-defined and known direction. Like the lead field, it represents a vector field with the property that a measure of the impact of a hypothetical dipole at an arbitrary point in the brain is obtained by calculating the scalar product with the respective dipole moment. While in the case of the lead field this measure of impact quantifies the contribution of a hypothetical dipole to the data recorded in a specific measurement channel, in the case of the resolution field it quantifies the contribution of a hypothetical dipole to the estimate of the AOI. The resolution-field concept, which uses elements of the Backus-Gilbert theory and is closely related to the concept of a resolution matrix, is illustrated with examples based on a simulated measurement with a 148-channel magnetometer system. © 1997 Elsevier Science Ireland Ltd.

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1. Introduction

Great efforts have been made in the last few years to develop advanced techniques for tracing back the electroencephalogram (EEG) and the magnetoencephalogram (MEG) to their underlying sources in the brain. It is possible now, for example, to work with relatively realistic volume conductor models of the human head (Hämäläinen and Sarvas, 1989; Bertrand et al., 1991; Thevenet et al., 1991; Yan et al., 1991) and with source models taking into account the real surface of the cortex, reconstructed from magnetic resonance tomograms (Dale and Sereno, 1993; Fuchs et al., 1994; Lütkenhöner et al., 1995). However, despite all this progress the principal problem remains that the estimation of sources in the brain from electrical or magnetic measurements outside the head represents an inverse problem which has no unique solution.

The only way to overcome the non-uniqueness of the inverse problem is to introduce additional constraints. In the ideal case, these constraints are derived from reliable a priori information about the sources, either obtained with other techniques or based on fundamental neurophysiological or neuroanatomical knowledge. In practice, however, such information is often not available so that it is required to be content with plausible assumptions. An example is the minimum-norm estimation (Hämäläinen and Ilmoniemi, 1984; Hämäläinen and Ilmoniemi, 1994). From the physiological viewpoint, the technique corresponds to the assumption that, given a set of source configurations able to explain the measured data, the configuration with the least energy consumption is the most plausible one.

As long as EEG and MEG are considered relatively independent from functional imaging techniques like positron emission tomography (PET) or functional magnetic resonance imaging (fMRI), the task of source analysis is to retrieve information not only about the activities of the contributing generators, but also about their locations. Unfortunately, spatial resolution is a weak point of EEG and MEG source analysis (Nunez, 1986; Wikswo and Roth, 1988; Tan et al., 1990; Lütkenhöner, 1991), and erroneous conclusions about the spatial structure of the source discredit other conclusions derived from the data. A way to overcome this problem could be the combination of EEG and MEG with PET or fMRI. The latter techniques have a better spatial resolution, but a relatively poor temporal resolution. In principle, such a synthesis opens the

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possibility to balance the shortcomings of the one group of methods by the strengths of the other group of methods. In the present case this would mean that the task of fMRI and PET would be to provide information about the source configuration, while the task of EEG and MEG would be to provide the time courses of the respective activities.

Efforts to combine EEG and MEG with functional imaging techniques are certainly steps in the right direction for the solution of many fundamental questions. However, the idea developed above represents an idealization, and it is to be expected that attempts to realize this idea in practice will lead to some serious problems. For example, the problem that EEG and MEG have a relatively poor spatial resolution remains, though in a weaker form, even if exact information about the source locations is available (Lütkenhöner, 1992). Furthermore, it is not guaranteed that brain regions giving rise to significant activities in PET or fMRI are generators of significant EEG or MEG activities as well, and, vice versa, a source significantly contributing to EEG and MEG may be invisible in PET and fMRI. The latter situation especially, would give rise to a big problem, because significant components of the measured EEG or MEG would inevitably be attributed to the wrong sources.

In conclusion, a priori information, if available, will not necessarily provide the complete spatial structure of the source model for the analysis of EEG and MEG so that it is important to explore strategies applicable also in the case of incomplete information. For that purpose it is useful to consider the following question: How to obtain the best estimate for the activity occurring in a given brain region, the region of interest, without having reliable information about the locations of other possible sources? The activity to be estimated shall be called the activity of interest (AOI). To understand the concept to be developed it is not essential to have a concrete idea of the AOI, but it may be helpful to consider it as the amplitude of a dipole at a specific location with well-defined and known direction (the examples given below are of this type). In principle, the AOI could represent any other reasonable measure of activity (e.g. the amplitude of a certain quadrupole component or the mean current density in an extended patch of the cortical surface).

The estimation of the AOI can be imagined as a projection of the measured data into the region of interest. Since there are concurrent activities at other locations, the task is to focus the data such that the AOI is reproduced as well as possible, while interferences from concurrent sources are minimized. This idea corresponds to the concept of a software lens developed by Freeman (1980).

An estimate for the AOI can be derived in many different ways so that it is important to have criteria for the assessment of the quality of a method. In what follows, a general concept for the comparison of different methods is developed. This concept can be applied completely independent of actual data. In contrast to previous work (Grave de Peralta Menendez et al., 1996), where data-independent figures of merits are introduced for an objective comparison of linear distributed inverse solutions, the theory developed here is applicable to any kind of linear inverse method, though at the expense that global aspects are pushed into the background.

To forestall misunderstandings it must be said that the theory described below will not be very useful in situations where one single equivalent current dipole or a small number of current dipoles (multi-dipole model) provide an almost perfect fit for the measured EEG or MEG. In such favorable cases, the optimal model is already available so that there is no need to consider other models. The situation changes, however if the signal-to-noise ratio is poor owing to concurrent sources in the brain (for example, when considering single epochs of event-related activity) or if the sources have a diffuse character so that a multi-dipole model is inappropriate. These are conditions where the concept described below can be helpful to select a method optimally adapted to the intentions of an investigator.

2. Theory

2.1. The resolution field

The starting point for the considerations that follow is the assumption that an estimate $\hat{a}$ of the AOI can be obtained by linearly combining the time functions recorded in the N measurement channels (EEG, MEG or a combination of both). Denoting these time functions as $d_i(t)$, $1 \leq i \leq N$, the above assumption can be written as:

$$\hat{a}(t|\mathbf{w}) = \sum_{i=1}^{N} w_i d_i(t) \tag{1}$$

The coefficients w_i are weighting factors which are dependent on the nature of the respective measurement channel and on the specific type of estimation performed. Examples will be given below. It is convenient to combine the set of weighting factors to a column vector $\mathbf{w}$. The dependency of the estimate $\hat{a}$ on $\mathbf{w}$ is indicated by using the notation $\hat{a}(t|\mathbf{w})$.

In practice, it is not to be expected that the generator of the AOI is the only source contributing to the measured EEG or MEG. Assume that an additional source is represented by a dipole at location $\mathbf{r}$ with moment $\mathbf{q}(t)$. The contribution of this hypothetical dipole to the data recorded in the ith channel can be written as $\mathbf{q}(t)\cdot\mathbf{f}_i(\mathbf{r})$, where $\mathbf{f}_i(\mathbf{r})$ is the lead field (Cuffin and Cohen, 1979; Hämäläinen et al., 1993; Malmivuo and Plonsey, 1995; Williamson and Kaufman, 1987) associated with the ith channel. This contribution is passed on to the estimate of the AOI, $\hat{a}(t|\mathbf{w})$, so that, according to Eq. (1), the total contribution of the hypothetical dipole at $\mathbf{r}$ to $\hat{a}(t|\mathbf{w})$ is $\mathbf{q}(t)\cdot\mathbf{R}(\mathbf{r}|\mathbf{w})$ with

$$\mathbf{R}(\mathbf{r}|\mathbf{w}) = \sum_{i=1}^{N} w_i \mathbf{f}_i(\mathbf{r}) \tag{2}$$

The vector $\mathbf{R}(\mathbf{r}|\mathbf{w})$ shall be called the resolution field since it is closely related to the concept of a resolution matrix, which plays an important role in the theory of discrete inverse problems. The continuous analogue of the resolution matrix, called the averaging function or resolving kernel, is an integral part of the Backus-Gilbert approach for the solution of inverse problems (Backus and Gilbert, 1968; Backus and Gilbert, 1970).

The resolution field is a vector field similar to the lead field. Lead field and resolution field have in common that, by calculating the scalar product with the dipole moment $\mathbf{q}$, a measure of the impact of a hypothetical dipole at $\mathbf{r}$ is obtained. However, the meaning of this measure of impact is different for the two fields: in the case of the lead field it quantifies the contribution of a hypothetical dipole to the data recorded in a specific measurement channel, whereas in the case of the resolution field it quantifies the contribution of a hypothetical dipole to the estimate of a specific AOI. Thus, the resolution field can be considered as the lead field of a synthetic virtual sensor (Robinson, 1989) providing the AOI.

To make it absolutely clear it shall be emphasized that the lead field as well as the resolution field are defined irrespective of the existence of a dipole at a certain location. They measure the *potential* impact of a dipole. With regard to the resolution field this means: supposed at location $\mathbf{r}$ there is a dipole with moment $\mathbf{q}$, then $\mathbf{q}\cdot\mathbf{R}(\mathbf{r}|\mathbf{w})$ is the contribution of this dipole to the estimate of the AOI. The resolution field does not judge this contribution, i.e. the contribution can be desired (if the dipole belongs to the source configuration responsible for the AOI) or undesired (if the dipole produces concurrent activity or noise). In the ideal case, the resolution field would be non-zero only in the vicinity of source elements belonging to the generator of the AOI. This is, unfortunately, not possible in practice, as will be illustrated later.

The above consideration for a single dipole can be easily transferred to arbitrary current distributions $\mathbf{J}$. The contribution of such a current distribution to the estimate $\hat{a}(t|\mathbf{w})$ is given by the volume integral

$$\hat{a}_J(t|\mathbf{w}) = \int_V \mathbf{J}(\mathbf{r},t)\cdot\mathbf{R}(\mathbf{r}|\mathbf{w})\mathrm{d}V \tag{3}$$

A similar equation relates the signal recorded in one of the channels to the respective lead field.

To calculate the resolution field, two components are required: the lead field vectors $\mathbf{f}_i(\mathbf{r})$ for the N measurement channels, and the weighting factors w_i. The latter will be considered in great detail below. The lead field depends on the measurement device and the volume conductor. In this study, only measurements with magnetometers will be considered, and the volume conductor is a homogeneous sphere. Furthermore, the magnetometer coils have a radial orientation. The lead field for this simple case is given by the equation

$$\mathbf{f}_i(\mathbf{r}) = \frac{\mu_0/4\pi}{|\mathbf{r}_i-\mathbf{r}|^2}\left(\frac{\mathbf{r}_i-\mathbf{r}}{|\mathbf{r}_i-\mathbf{r}|}\times\frac{\mathbf{r}_i}{|\mathbf{r}_i|}\right) \tag{4}$$

where $\mathbf{r}_i$ denotes the location of the ith coil.

2.2. The weighting factors

The weighting factors w_i are highly dependent on the estimation procedure used. This shall be illustrated with four examples: the best channel-estimator, the least-squares estimator, the maximum-likelihood estimator, and the minimum-norm estimator.

To begin with a very simple example, it shall be assumed that there is only one single active source in the brain so that all channels reflect the time course of this source, except for superposed noise. Thus, disregarding a constant scaling factor, the time course obtained for the channel with the best signal-to-noise ratio (the 'best' channel) can be used to estimate the time course of the source. With regard to the resolution-field concept introduced above this means that all weighting factors w_i are zero, except for the weighting factor corresponding to the best channel. If this is the kth channel, Eq. (2) reduces to $\mathbf{R}(\mathbf{r}|\mathbf{w}) = w_k\mathbf{f}_k(\mathbf{r})$, i.e. the resolution field differs only by a constant factor from the lead field corresponding to the best channel.

If the recorded data are noisy, it is advisable to estimate the source activity on the basis of *all* channels rather than to focus on the best channel. This means that a weighted average of all measured time functions has to be calculated according to Eq. (1). This can be achieved, for example, by fitting the model of a current dipole with fixed location and fixed orientation to the data, because, given the latter parameters, the remaining optimization problem for the amplitude of the dipole moment has a solution consistent with Eq. (1). Two different optimization procedures for the amplitude of the dipole moment will be considered: a least-squares fit and a maximum-likelihood estimation (Sekihara et al., 1992).

The assumption of one single dipole is questionable in many (if not most) practical applications. An alternative is the assumption of many (typically a few hundred) dipoles homogeneously distributed in the brain. The attempt to estimate the moments of these dipoles generally results in an underdetermined problem, i.e. there are more model parameters than measurement channels. To overcome this problem, additional constraints are required. Very common is the *minimum-norm* estimation already mentioned in the introduction (Hämäläinen and Ilmoniemi, 1984; Hämäläinen and Ilmoniemi, 1994). The solution of the minimum-norm estimation problem results in the dipole moments not only at the point of interest, but in all the other locations as well. To focus on the activity of interest, the dipole moment obtained for the point of interest is projected onto the direction of interest, while the information obtained for the other locations is simply discarded.

2.3. The normalized resolution field

The scale of the resolution field is not necessarily well-defined, owing to the fact that there is some arbitrariness with regard to the scaling of the weighting factors. This causes a problem if resolution fields based on different assumptions are to be compared. To avoid this problem, a normalization is introduced as follows: a resolution field is called a normalized resolution field if the vector of weighting factors, **w**, fulfills the condition

$$\mathbf{w} \cdot \mathbf{d}^{AOI} = 1 \tag{5}$$

where $\mathbf{d}^{AOI}$ is the data vector arising from an AOI with amplitude one. A consequence of this kind of normalization is that the AOI is retrieved without error if there are no other sources in the brain and the sensors (electrodes or magnetometer coils) pick up no noise at all. In what follows, only normalized resolution fields will be considered so that explicit mention of the normalization is dispensable. In some of the examples considered here the set of weighting factors is normalized from the outset: if not, they are assumed to be rescaled appropriately.

2.4. Variance of estimated AOI

If the recorded data are composed of a signal and additive noise having a Gaussian amplitude distribution, with no correlation between signal and noise, then it is straightforward to derive a simple formula for the variance of the estimator $\hat{a}(t|\mathbf{w})$:

$$\mathrm{var}[\hat{a}] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i c_{i,j} w_j \tag{6}$$

The quantity $c_{i,j}$ in this equation specifies the covariance of the noise picked up by the channels i and j.

3. Examples

The theoretical considerations presented above shall be illustrated now with a few examples. The model configuration is identical in all these examples, but the strategy used to estimate the AOI differs.

3.1. Model

The data are assumed to be recorded with a 148-channel magnetometer system characterized by radially oriented coils located on a spherical surface with a radius of 12 cm (Lütkenhöner et al., 1996). Brain and scalp are assumed to be represented by concentric spheres with 8 cm and 10 cm radius, respectively. The AOI corresponds to the amplitude of a dipole located on the z-axis, having a distance of 6 cm from the center of the sphere and pointing into the x direction. Thus, in the present case, the region of interest is represented by a discrete point in the brain: the location of the dipole whose moment is to be estimated. This location shall be denoted as the location of interest.

Two different noise models are considered: spatially uncorrelated noise having a Gaussian amplitude distribution with variance one, and spatially correlated noise arising from random dipoles in the brain. For the sake of simplicity, the two types of noise will be referred to as the uncorrelated and the correlated noise. The former model is not very realistic, because EEG and MEG components not correlated with the AOI ('noise') are usually dominated by spatially correlated contributions from concurrent activities in the brain, not by instrumental noise (Wikswo et al., 1993; Kuriki et al., 1994). Nevertheless, the model of spatially uncorrelated noise is important since it is the model underlying the widely used least-squares estimation technique. The idea underlying the more realistic random dipole model is that the brain is homogeneously filled with dipoles having random orientations and amplitudes (Cuffin and Cohen, 1977; Lütkenhöner, 1991; de Munck et al., 1992). The covariances for this model were calculated using an analytical formula derived for the case that each random dipole has three independent components with a Gaussial amplitude distribution (Lütkenhöner, 1994). To account for the fact that measured data typically contain also spatially uncorrelated instrumental noise, the diagonal elements of the noise covariance matrix were multiplied by the factor 4/3. This corresponds to the assumption that the variance of the spatially uncorrelated noise is one third of the variance of the spatially correlated noise arising from the random dipoles. The covariance matrix was finally rescaled so that its diagonal elements had the value 1. This means that the uncorrelated and the correlated noise had the same variances.

3.2. Resolution field

The resolution field represents a vector field defined for each point in the brain. However, for visualization purposes it is useful to consider only a relatively small sample of representative points. In the present study, points inside the sphere representing the brain where taken from a rectangular grid (2-cm spacing between adjacent grid lines). The resolution field was calculated for each of the resulting 257 points, and the results were plotted as arrows, except that arrows corresponding to a resolution field with a length smaller than 1/4 were suppressed.

A visualization of the resolution field for the best-channel estimator is displayed in the upper row of Fig. 1. The left panel shows a view from the left, whereas the right panel shows a view from the top. The best channel corresponds to the coil plotted in red. The resolution-field vector obtained for the point of interest (red arrow) has the length $|\mathbf{R}| = 1.035$, which differs little from the *ideal* value for the resolution field vector at the point of interest.[1] The figure demonstrates that the best-channel estimator is not at all specific for the point of interest. There are many other locations for which the estimate of the AOI is even more sensitive.

A resolution field with a completely different appearance is obtained if a least-squares estimator rather than the best-channel estimator is used (middle row of Fig. 1). The pattern of resolution field vectors is symmetrical with regard to the x–z plane and anti-symmetrical with regard to the y–z plane. The length of the resolution-field vector at the location of interest (red arrow) has now the ideal value $|\mathbf{R}| = 1$. Among the source locations considered there is only one location where $|\mathbf{R}|$ is greater than at the location of interest: this is the location closest to the measurement locations, which has the Cartesian coordinates (0,0,8). The length of the resolution-field vector at this location is given in the fourth column of Table 1. It is not very surprising that, for the least-squares estimator considered here, $|\mathbf{R}|$ tends to increase with decreasing distance between source and coils because the amplitude of the magnetic field decreases with the square of this distance. Though this effect is partially compensated for by the fact that the number of coils significantly contributing to the estimate of the AOI diminishes with decreasing distance (Lütkenhöner, 1996), it is clear that, in order to have a specified impact on the estimated activity, sources near the surface of the brain require a smaller dipole moment than deeper sources.

The resolution-field pattern obtained for the least-squares estimator is certainly highly superior to that obtained for the best-channel estimator. Nevertheless, the pattern is still unsatisfactory, because the specificity for the AOI is not very pronounced. A much higher specificity is obtained for the maximum-likelihood estimator (bottom row of Fig. 1). The respective weighting factors were derived using the correlated-noise model. Compared to the previous examples, the resolution-field pattern obtained for the maximum-likelihood estimator is clearly more compact, and it is tempting to assume that the maximum-likelihood estimator is highly superior to the two estimators considered before. However, even if the maximum-likelihood estimator may have a better performance in most practical situations, superiority is not guaranteed. The maximum-likelihood estimator is definitely the optimal choice if the model assumptions underlying the construction of the estimator are perfectly fulfilled. If this is not the case, however, other estimators may perform better. Suppose, for example, that the dominating noise source is located at the intersection of the z-axis and the surface of the brain, i.e., at the position (0,0,8). As in the case of the least-square estimator, this position, having the closest distance to the measurement surface, is associated with the longest resolution-field vector. This means that a dipole at this position is capable of affecting the estimate of the AOI with the least effort. The fourth column of Table 1 shows that, at this location, the maximum-likelihood estimator has a resolution-field vector which is longer than that of the least-squares estimator. Thus, supposed there is an exceptionally strong noise source at this location (which would violate, of course, the assumptions underlying the construction of the maximum-likelihood estimator), then the least-squares estimator will be superior.

The images obtained for the minimum-norm estimator are not presented here because they turned out to be almost indistinguishable from those presented in the bottom row of Fig. 1. Regarding the fact that the calculations of maximum-likelihood and minimum-norm estimator are so different, it is a remarkable finding that the resolutions fields of these two estimators are so similar.

3.3. Weighting factors

The similarities of the resolution-field patterns obtained for the maximum-likelihood estimator and the minimum-norm estimator give rise to an important question: are these two estimators very similar also in other important respects or is the similarity basically confined to the resolution field? To answer this question it is instructive to consider the outcome of an intermediate step in more detail: the weighting factors for the individual channels.

The weighting factors used in the above examples are illustrated in Fig. 2. The 148 coil locations were projected into a plane, and the values of the respective weighting factors were visualized by controlling type and size of the symbol plotted. A weighting factor is represented by a dot if its value is small compared to the largest weighting factors, otherwise it is represented by a circle with an area corresponding to its absolute value. A filled circle indicates a weighting factor with negative sign. The pattern obtained for the best-channel estimator (Fig. 2a) is, of course, trivial and does not require any further comment. The patterns obtained for the least-squares estimator (Fig. 2b) and the maximum-likelihood estimator (Fig. 2c) are similar: positive weighting factors in the upper half of the plot ($y > 0$), and negative weighting factors in the bottom half ($y < 0$). This result reflects the fact that, in the present

[1] Though the deviation from the ideal value one is quite insignificant, it is instructive to analyse the reason for this discrepancy. For the geometry considered in this example, the dipole associated with the AOI produces a magnetic field pattern with the extrema located on the y-axis. However, owing to the discrete nature of the measurement, the best-channel corresponds to a measurement location lying slightly apart from the y-axis. Since the resolution-field concept provides an objective measure and is not influenced by the intentions of the user, it reflects this small inconsistency: a measurement coil located near the y-axis (but not located exactly on this axis) is most sensitive for a dipole forming a small angle with the x-axis. As a consequence, the resolution-field vector at the point of interest is not completely aligned with the x-axis, and the normalization according to eqn (5), where a dipole oriented exactly in x direction is assumed, yields a length slightly greater than 1.

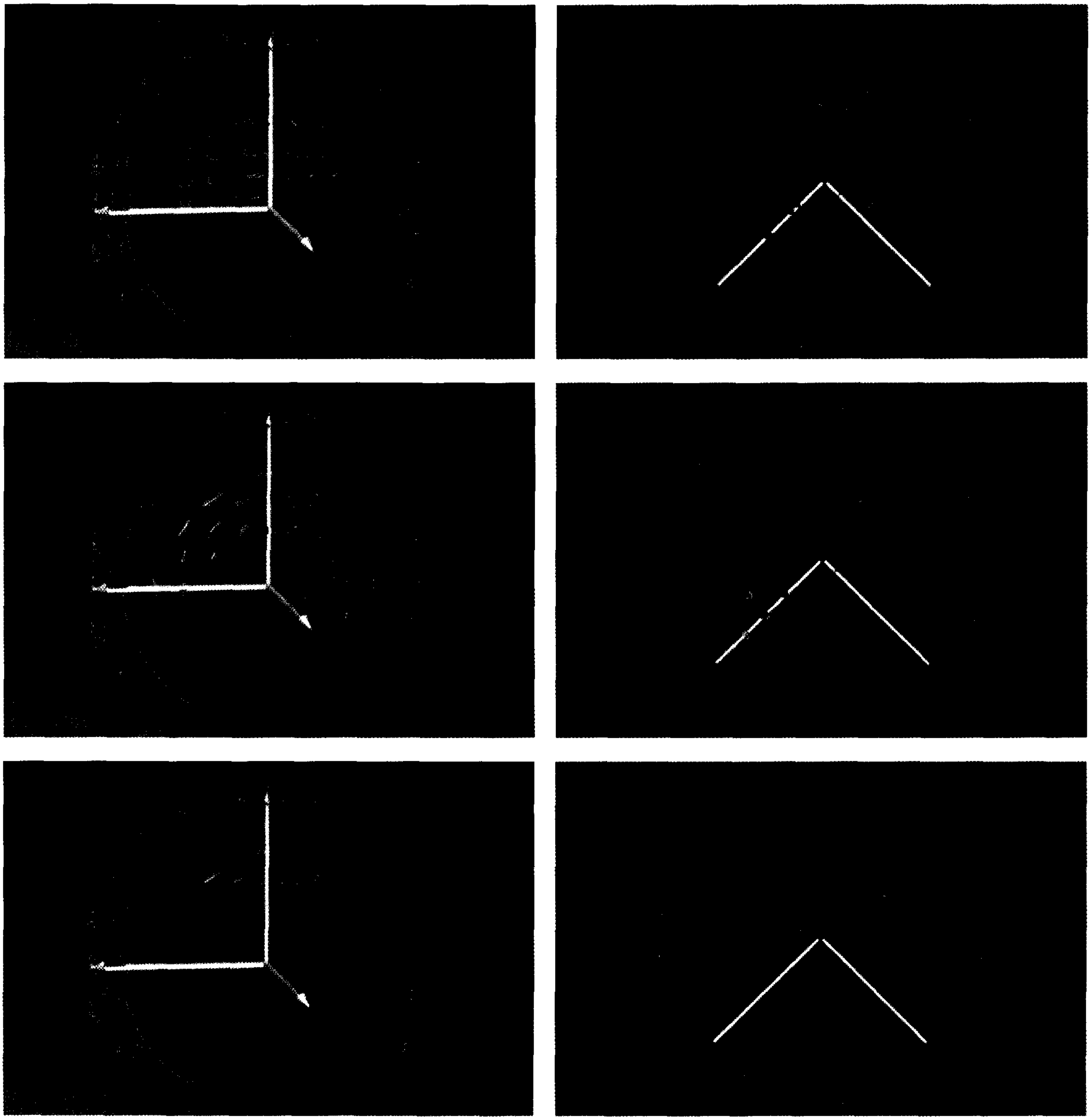

Fig. 1. Resolution fields obtained for the best-channel estimator (upper row), the least-squares estimator (middle), and the maximum-likelihood estimator (bottom). The left column shows views from the left, the right column views from the top. To allow a free view to the dipoles (arrows) visualizing the resolution field, part of the coils were removed, and the spheres representing scalp and brain were opened (left column: left hemispheres removed; right column: upper hemispheres removed). In the left column, the dipoles in the left hemisphere were removed as well, to improve the visibility of the dipoles in the right hemisphere. In the case of the best-channel estimator (upper row), the coil representing the best channel is plotted in red.

case, the AOI corresponds to a dipole located on the z-axis, with its moment pointing in x direction. This means that for measurement locations, with a positive y coordinate the magnetic field has the same sign as the amplitude of the dipole moment, whereas an opposite sign is obtained for coils with a negative y coordinate. Thus, to obtain an estimate for the AOI, contributions from coils with a negative y coordinate have to be multiplied with a negative weight. A striking difference between least-squares and maximum-likelilood estimates is that the latter is essentially based on only a few channels close to the central coil (z axis). This result suggests that, for the source considered here, magnetometer systems covering only a limited area of the scalp have about the same performance as a whole-

Table 1

Variance measure var[$\hat{a}$] for the correlated noise (second column) and the uncorrelated noise (third column) as well as amplitude of resolution-field vector obtained for the point closest to the measurement coils (fourth column)

Estimator	var[$\hat{a}$] for different types of noise		$|\mathbf{R}(0,0,8|\mathbf{w})|$
	Uncorrelated	Correlated	
Best-channel	0.0201	0.0201	2.11
Least-squares	0.0008	0.0122	1.66
Maximum-likelihood	0.0036	0.0055	2.72
Mimmum norm	0.0936	0.0281	2.78

The variance measures have the unit $(\mathrm{nAm})^2$, provided that the noise registered by the measurement coils has the standard deviation 1 fT. If the noise has a different srandard deviation, the variance measures have to be rescaled accordingly.

head magnetometer system, provided that a maximum-likelihood estimator is used.

A completely different pattern is obtained for the minimum-norm estimator. Fig. 2d shows that positive and negative weighting factors are intermixed. This means that the minimum-norm estimator performs complex difference operations with the data from the different channels. This is a remarkable result with regard to the fact that the resolution field of the minimum norm estimator is almost indistinguishable from that of the maximum-likelihood estimator.

The second column in Table 1 gives the sum of the weighting factors squared. In the case of uncorrelated noise with variance one this measure can be interpreted as the variance of $\hat{a}$, as can be deduced from Eq. (6). It is not surprising that, with regard to this measure, the least-squares estimator has the best performance, since the assumption of uncorrelated noise with identical standard deviations in all channels is the principle underlying the least-squares estimation procedure. The value obtained for the maximum-likelihood estimator is more than four times greater than that obtained for the least-squares estimator. Nevertheless, the maximum-likelihood estimator performs quite reasonably compared to the other two estimators. The noise sensitivity of the minimum-norm estimator is extremely bad. A stronger regularization would certainly reduce this sensitivity, but this is not the topic of the present paper.

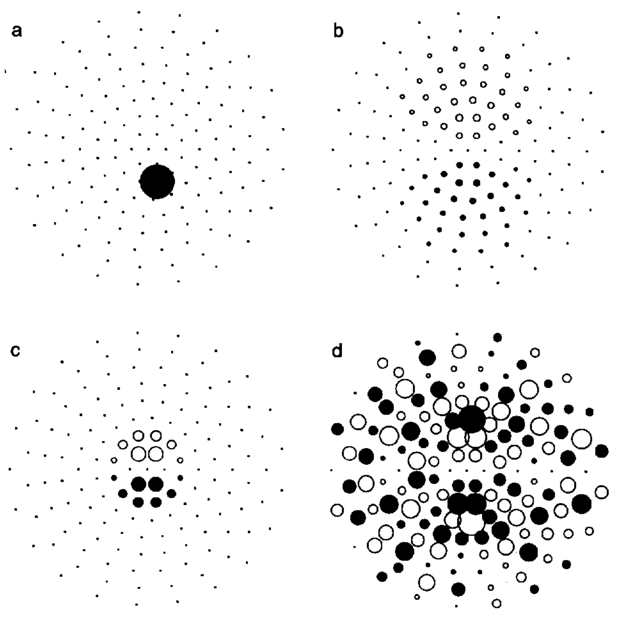

Fig. 2. Visualization of the weights for the individual channels. (a) Best-channels estimator. (b) Least-squares estimator. (c) Maximum-likelihood estimator. (d) Minimum-norm estimator. The coil locations were projected into a plane so that x- and y-axis of the three-dimensional space were mapped to abscissa and ordinate, respectively. The area of a symbol is proportional to the absolute value of the weighting factor for the respective coil, except that weighting factors with an absolute value smaller than a certain limit are represented by dots. Negative weighting factors indicated by filled symbols.

Eq. (6) allows one to calculate the variance of $\hat{a}$ for arbitrary correlations between channels. The third column gives the variances obtained for the correlated-noise model. As expected, it is the maximum-likelihood estimator which has the best performance now, whereas the variance obtained for the least-squares estimator is increased by more than 1 order of magnitude. It is remarkable that the minimum-norm estimator performs much better than for uncorrelated noise, though it does not perform as well as the least-squares estimator. The performance of the best-channel estimator is, of course, not dependent on assumptions about correlations between channels.

In a supplementary investigation, the variance of $\hat{a}$ was calculated for spatially correlated noise without an additional uncorrelated component (i.e., the multiplication of the diagonal elements of the noise covariance matrix by the factor 4/3 was omitted). The variances obtained for the maximum-likelihood and the minimum-norm estimator were 0.0049 and 0.0062, respectively. This result is of considerable importance. It suggests that even without having knowledge of the covariance matrix of the noise (such knowledge is a prerequisite for the application of the maximum likelihood estimation technique), a nearly optimal estimator (with respect to the signal-to-noise ratio) can be obtained just by applying the minimum-norm estimation procedure. This conclusion, which is valid, of course, only for the ideal case that all noise sources are located inside the brain, represents an interesting consistency check for the resolution-field concept: since the maximum-likelihood estimator and the minimum-norm estimator are associated with very similar resolution fields, Eq. (3) suggests that the variances of $\hat{a}$ should be similar as well, provided that there are no noise sources outside the volume V representing the brain. The above result confirms this expectation convincingly.

4. Discussion

For a proper interpretation of EEG and MEG it is useful to have a certain model in mind. In view of the complexity of human brain function it is inevitable that any model represents a great simplification. In practice, the situation can be even worse since the assumptions underlying the model can be oversimplified or even completely erroneous. For example, the model of a single equivalent current dipole would not be so common nowadays if its application were strictly confined to situations with just one active brain area. Another common modeling error is the postulation of a noise covariance matrix not necessarily reflecting the situation at the time of measurement (e.g. implicit assumption of spatially uncorrelated noise in the case of a least-squares estimation; use of a pre-stimulus covariance matrix for the analysis of post-stimulus data). But even if such errors are avoided and a custom-made model is used, it is not guaranteed that the model will do what it is supposed to do, because the estimation of the model parameters from the measured EEG or MEG represents an inverse problem, and the solution of this problem may not permit the intended conclusions. By applying the concepts developed in this communication it is possible to explore potential shortcomings of an analysis procedure in an a priori manner, i.e., without requiring actual data.

At first glance, the usefulness of the resolution-field concept may appear to be limited by the fact that the AOI is defined as an amplitude measure, whereas currents in the brain correspond to vectors. It would be no principal mathematical problem to generalize the resolution-field concept such that the AOI is allowed to be a vector. However, such a generalization does not seem to be helpful: instead of a resolution field some kind of resolution tensor would be obtained, which would be hard (if not impossible) to visualize. Furthermore, for currents in the cortex it is not a problem at all that the AOI represents an amplitude measure, because it is reasonable to assume that these currents are oriented perpendicular to the cortical surface. Thus, it is not required to estimate the direction of the currents from the measured EEG or MEG, provided that a reconstruction of the cortical surface from magnetic resonance images is available. Even if such information is missing it may be possible to associate a location of interest with a certain direction. For example, using a dipole source analysis of MEG data an investigator interested in the primary auditory cortex can define not only the location of the auditory cortex, but also the mean direction of the currents in this area.

As already indicated, the resolution-field concept uses elements of the Backus-Gilbert theory (Backus and Gilbert, 1968; Backus and Gilbert, 1970), with the resolution field being a special case of the resolution function (or averaging kernel) of Backus and Gilbert. This theory, originally developed in a geophysical context, was brought to the attention of the EEG and MEG community by Robinson (Robinson, 1989; Robinson and Rose, 1992). Taking the idea of the resolution field as a basis, the principle of the Backus-Gilbert approach is, roughly speaking, to construct an estimator with the property that the associated resolution field is as compact as possible, centered around the location of interest (minimization of the 'spread' of the resolution field). Thus, if the main criterion for the 'goodness' of an inverse procedure is the spread of the resolution field, then the Backus-Gilbert method is certainly the first choice. However, like any other method, the Backus-Gilbert method has not only advantages, but also serious drawbacks. The price to be paid for the achievement that the spread of the resolution field (or resolving function, to use the nomenclature of Backus and Gilbert) is minimal is that the method does not explain the data exhaustively (Menke, 1984). Furthermore, the numerical effort required for the computation of a complete solution (i.e., a solution for a sufficiently dense grid of points in the brain) is much higher than that typically required for other methods. For these reasons the Backus-Gilbert theory did not find the same attention as other inverse procedures for the analysis of EEG and MEG.

A conceptional difference between the Backus-Gilbert theory and the resolution-field concept as described here is that the primary purpose of the latter is to mediate an intuitive comprehension of the essential properties of a given estimator, whereas the Backus-Gilbert theory provides a complete framework for the construction of an estimator. After having used the resolution-field concept to get insight into a method, the idea may arise, of course, to improve, in some sense, the performance of the method. The resolution-field concept does not tell how to achieve this, but it provides a tool to assess the performance of a modification. The examples presented here have shown that the resolution-field concept is useful not only in the context of the Backus-Gilbert method, but can be applied to any other linear inverse procedure as well. While an inverse procedure is typically based on a global optimization concept, the resolution field focuses on local issues, namely a specific AOI. If there is more than one AOI or if it appears appropriate to consider a certain number of representative points in the brain, the resolution field can be calculated, of course, successively for all points of interest.

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